

TRACCIA 6/6/12

ESERCIZIO n. 2

$$X, Y \sim N(0, 2) \quad E[XY] = -1 \quad Z \sim N(3, 4)$$

~~Z~~ INDIPENDENTE DA X E Y

$$\begin{aligned} E[(2X - Z)^2] &= E[4X^2 + Z^2 - 4XZ] = E[4X^2] + E[Z^2] + E[-4XZ] \\ &= 4E[X^2] + E[Z^2] - 4E[X] \cdot E[Z]; \end{aligned}$$

POICHE' SONO
INDIPENDENTI

$$\begin{aligned} \text{Var}(X) &= E[X^2] - E[X]^2 \Rightarrow E[X^2] = \text{Var}(X) + E[X]^2 \Rightarrow \\ E[X^2] &= 2 + (0)^2 = 2 \end{aligned}$$

$$E[Z^2] = \text{Var}(Z) + E[Z]^2 = 4 + (3)^2 = 13 \Rightarrow$$

$$\begin{aligned} 4E[X^2] + E[Z^2] - 4E[X] \cdot E[Z] &= \\ = 4(2) + 13 - 4(0 \cdot 3) &= 8 + 13 - 0 = 21 \end{aligned}$$

$$\text{Var}(1 + X - 2Y + Z) = \text{Var}(X - 2Y) + \text{Var}(Z)$$

POICHE' Z INDIPENDENTE
DA X E Y.

$$\text{Var}(1) = 0$$

$$\begin{aligned} \text{Var}(X - 2Y) &= \text{Var}(X) + \text{Var}(-2Y) + 2(E[X \cdot (-2Y)] - E[X] \cdot E[-2Y]) = \\ &= 2 + 4(2) + 2(-2E[X \cdot Y] - (0 \cdot (-2 \cdot 0))) = \\ &= 2 + 8 + 2(-2(-1) - 0) = 2 + 8 + 4 = 14 \Rightarrow 14 + \text{Var}(Z) = 14 + 4 = 18 \end{aligned}$$